Homework 2

COMS W3261, Summer B 2021

This homework is due **Monday, 7/12/2021, at 11:59PM EST**. Submit to GradeScope (course code: X3JEX4).

Grading policy reminder: LaTeX is preferred, but neatly typed or handwritten solutions are acceptable. Your TAs may dock points for indecipherable writing. Proofs should be complete; that is, include enough information that a reader can clearly tell that the argument is rigorous.

Remember that the tool http://madebyevan.com/fsm/ may be useful for drawing finite state machines.

1 Problem 1 (7 points)

Evaluate each of the following regular expressions and write down the language it describes as a set or with a single sentence. (Example: $01^+ = \{w \mid w$ consists of a single 0 followed by at least one $1\}$ or "This regular expression describes the language of strings that consist of a single 0 followed by at least one 1".)

1. (1 point.) Let $\Sigma = \{0,1\}$. Evaluate $0\Sigma^*1\Sigma^*0$.

2. (1 point.) Evaluate $(01 \cup 0 \cup 1)0^*(1^+)\emptyset$.

   Write regular expressions that evaluate to the languages given.

3. (1 point.)

   $\{w \mid w$ consists of a (non-empty) substring of a's and b's of even length, followed by the substring '01'.$\}$

   Note: an earlier version of this question didn't specify that the substring of a's and b's was non-empty, so we will give credit for either interpretation.

4. (1 point.)
   $$\{w \mid w \text{ is a string of 0's with length divisible by } 2,3,5, \text{ or } 7.\}$$

Evaluate each of the following regular expressions and write down the language it describes as a set or with a single sentence.

5. (1 point.) Let $H = \{0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F\}$, and let $\Sigma = H \cup \{\#\}$. Evaluate $\#HHHHHH$.

6. (1 point.) Let $P = \{K,Q,R,B,N\}$, $X = \{a,b,c,d,e,f,g,h\}$, $Y = \{1,2,3,4,5,6,7,8\}$. Evaluate $(P \cup \varepsilon)(\times \cup \varepsilon)XY$.

7. (1 point.) Evaluate the regular expression

$$\rightarrow (\bigcirc(\underset{0}{\rightarrow} \cup \underset{1}{\rightarrow}))^* \odot.$$

   (The alphabet is $\Sigma = \{\rightarrow, \underset{0}{\rightarrow}, \underset{1}{\rightarrow}, \bigcirc, \odot\}$.)

2   Problem 2 (6 points)

1. (6 points). Draw a state diagram for an NFA **with at most 3 states** that recognizes the regular expression

$$((1 \circ 10) \cup 11)^*.$$

Explain in words why your NFA recognizes the language specified.

3 Problem 3 (6 points)

1. (6 points.) Given a language A, define the language $\mathrm{pre}(A)$ as follows:

$$\mathrm{pre}(A) := \{xy \mid x \in A\}.$$

Prove that the class of regular languages is closed under $\mathrm{pre}(\cdot)$.

Note: the '·' in $\mathrm{pre}(\cdot)$ refers to a generic function argument - we're just proving that the class of regular languages is closed under pre. y can be any string over the same alphabet as A.

4 Problem 4 (1 point)

1. What is one thing the instructor or course staff could do better to make the material or expectations clearer or more convenient for you?

2. Adjusting for the compressed timeframe of this course (two weeks in one), how are you finding the difficulty so far in relation to other CUCS courses? (For example: 'much easier', 'somewhat easier', 'about the same', 'somewhat harder', 'much harder'.)

3. (Optional) Any other thoughts? Thank you!