

# Homework 3

COMS W3261, Summer B 2021

This homework is due **Monday, 7/19/2021, at 11:59PM EST**. Submit to GradeScope (course code: X3JEX4).

Grading policy reminder:  $\text{\LaTeX}$  is preferred, but neatly typed or handwritten solutions are acceptable. Your TAs may dock points for indecipherable writing. Proofs should be complete; that is, include enough information that a reader can clearly tell that the argument is rigorous.

The tool <http://madebyevan.com/fsm/> may be useful for drawing finite state machines.

If a question is ambiguous, please state your assumptions. This way, we can give you credit for correct work. (Even better, post on Ed so that we can resolve the ambiguity.)

## 1 Problem 1 (12 points)

1. (6 points.) Prove that the language

$$A = \{w \mid \text{For all } y \in \{0,1\}^*, w \neq yy\}$$

over the alphabet  $\Sigma = \{0,1\}$  is nonregular. You may use the pumping lemma and/or closure properties.

2. (6 points.) Prove that the language

$$B = \{1^n 0^m 1^n \mid n \geq 0, m \geq 1\}$$

over the alphabet  $\Sigma = \{0,1\}$  is nonregular. You may use the pumping lemma and/or closure properties.

## 2 Problem 2 (8 points)

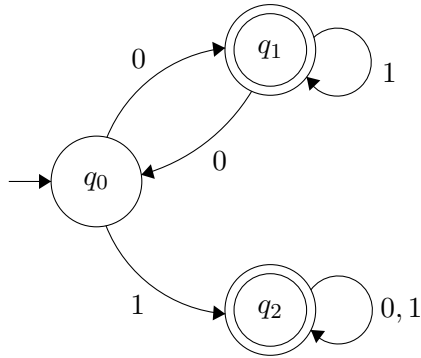
1. (8 points.) Is the language

$$C = \{a^i b^j c^k \mid i, j, k \geq 0; i \leq j\} \cup \{a^i b^j c^k \mid i, j, k \geq 0; j \leq k\} \cup \{a^i b^j c^k \mid i, j, k \geq 0; k \leq i\}$$

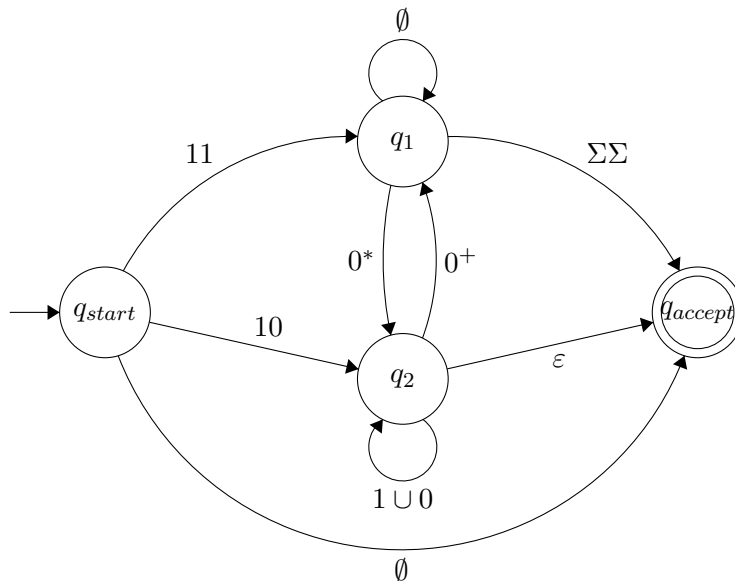
over the alphabet  $\Sigma = \{a, b, c\}$  a regular language? Prove your answer.

### 3 Problem 3 (10 points)

- (4 points). Convert the DFA below into a GNFA state diagram using the procedure outlined in class. (This procedure is also outlined in the textbook on page 71.)



- (6 points). Use the procedure  $CONVERT(G)$  outlined in class (and on page 73 of the textbook) to compute the values of the transitions  $\delta'(q_{start}, q_2)$ ,  $\delta'(q_{start}, q_{accept})$ , and  $\delta'(q_2, q_{accept})$  after removing state  $q_1$  from the GNFA below. Hint: Recall that  $\emptyset^*$  evaluates to the language  $\{\varepsilon\}$ .



#### 4 Problem 4 (12 points)

1. (3 points). What is the language of the grammar  $G_1$  below? Here  $S$ ,  $A$ , and  $B$  are the variables and 0 and 1 are the terminals. Explain your reasoning.

$$\begin{aligned}S &\rightarrow 1A1 \\A &\rightarrow S \mid B \\B &\rightarrow 0B \mid \varepsilon\end{aligned}$$

2. (3 points). What is the language of the grammar  $G_2$  below? Here  $A$  and  $B$  are the variables and  $x$ ,  $y$ , and  $z$  are the terminals. Explain your reasoning.

$$\begin{aligned}A &\rightarrow xAx \mid yAy \mid zAz \mid B \\B &\rightarrow x \mid y \mid z \mid \varepsilon\end{aligned}$$

3. (3 points). Design a grammar for the language

$$D = \{a^i b^j c a^j b^i \mid i, j \geq 1\}$$

and explain why your grammar produces  $D$ .

4. (3 points). Design a grammar for the language

$$L = I(\text{saw} \cup \text{met} \cup \text{loved})(\text{the} \cup a)(\text{very})^*(\text{large} \cup \text{tiny} \cup \text{red})(\text{frog} \cup \text{dog})$$

and explain why your grammar produces  $L$ . (You can treat each word as a single terminal symbol.)