COMS W3261 - Lecture 0

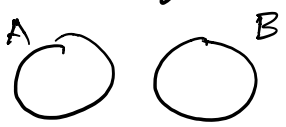
Sipser - Intro to the Theory of Computation, 3rd ed.

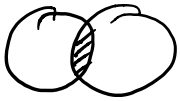
Chapter 0 - 0.2

set : bag of unique objects

$A = \{1, 2, 3\}$ $B = \{a, b, c\}$

multiset : not unique. $\{1, 1, 2\}$




$A \cap B$


$A \oplus B$


$A \cup B$


$A \setminus B$

$a \in B$ $C \subseteq B$

$a \notin A$ $C \subset B$

$|A| = 3.$

Sequence := ordered set.

$C := (a, b, c)$ $E := (a, a, b)$

$D := (b, a, c)$ $F := (1, 2, 4, 8, 16, \ldots)$

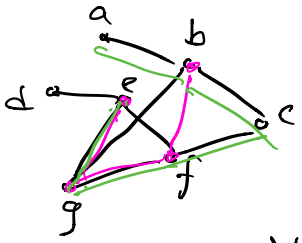
finite? Call it a k-tuple.

$\times$ - operator: Cartesian product.

$\{a, b\} \times \{1, 2\} = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$\mathbb{Z}^2$, $\mathbb{Z} \times \mathbb{Z} = \{(\ ) \ldots\}$

GRAPHS:



$G := (V, E)$

V: set of vertices; E: set of edges

$V = \{a \dots g\}$

$E = \{(a,b), (b,c), (c,f) \dots\}$

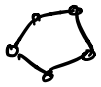
$\deg(v) :=$ number of incident edges

$\deg(e) = 3$

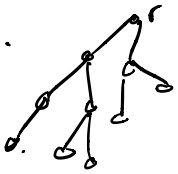
subgraph := graph where, $H = (V_2, E_2)$ if $V_2 \subseteq V$, $E_2 \subseteq E$

path := (e, g, f, c, b, a)

cycle := path that ends where it starts.



tree := connected graph with no cycles.



directed graph (digraph):

(a,b)

(b,a)

STRINGS & LANGUAGES

alphabet: set of characters/symbols.

$\Sigma = \{0, 1\}$

$= \{a, b, c \dots z\}$ ñ ø ä

String: finite sequence of symbols.

0110110 tim

Language: set of strings.

'English' := {all words w: w is in the English language}

$L := \{\text{all strings } w \in \{0,1\}^n : w \text{ is a palindrome}\}$

n = 3 000, 010, 101

Boolean Logic.

True = T = 1

False = F = 0

$\wedge$ — and

$\vee$ — or

$\neg$ — not

$\rightarrow$ — implies.

$\otimes$ — xor

$1 \wedge 1 = 1$

$1 \oplus 0 = 1$

$1 \oplus 1 = 0$

$((1 \vee 0) \vee 1) \rightarrow 1$

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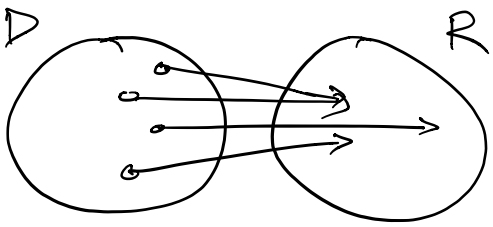
functions $f: D \rightarrow R$



Proofs:

Construction —

Contradiction —

Induction —