COMS W3261 — Lecture 2, Part 2:

~ Nondeterminism ~

Deterministic Finite Automata

next state completely determined by the current state.

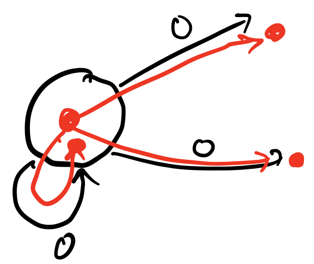
$$\delta : Q \times \Sigma \to Q.$$

Idea: What if we break determinism?

New rules for Nondeterministic Finite Automata (NFAs):

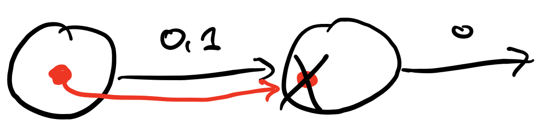
1. Multiple options from a state for one symbol? Take all of them. (Split into two branches that run in parallel.)

On 0:



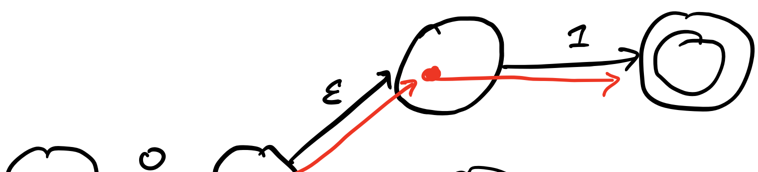
2. No options on a symbol? This branch "dies." (All branches are dead: reject.)

Read in: 01:



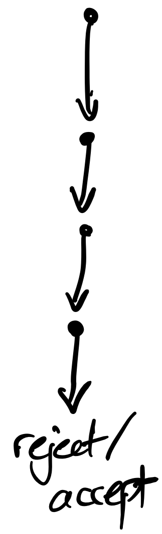
3. ε-arrows indicate an extra "free branch." This resolves after each computational step.

On 01:

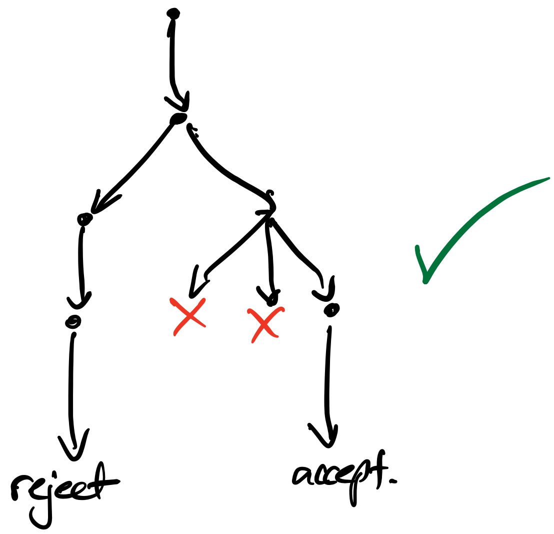


4. <u>Accept</u> if any live branch of computation accepts at the end of the input string.

<u>Deterministic:</u>

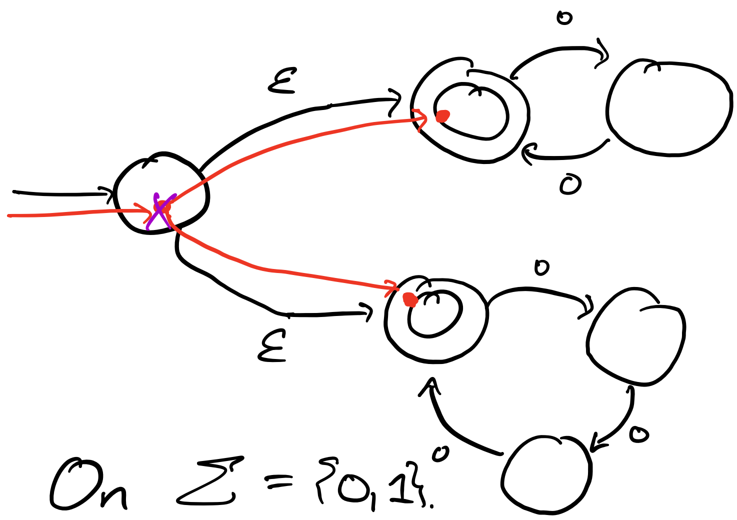


<u>Nondeterministic:</u>



Example 1. On $\Sigma = \{0\}$.

Goal language: $\{ w \mid |w| \text{ is divisible by } 2 \text{ or by } 3 \}$ — length/size

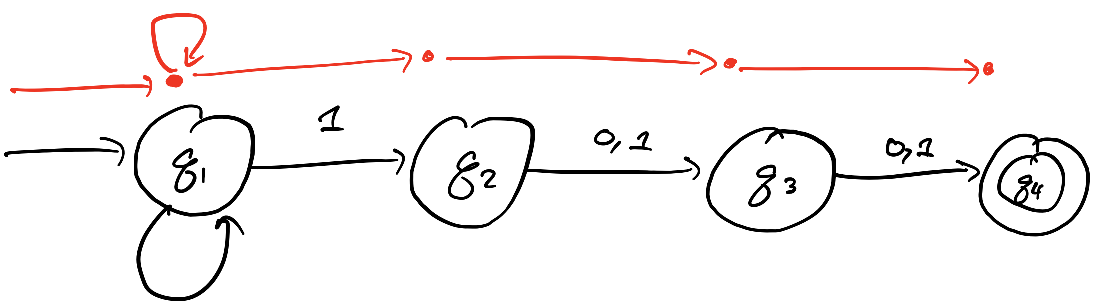


// recognize $|w|$ divisible by 2.

// recognize $|w|$ divisible by 3

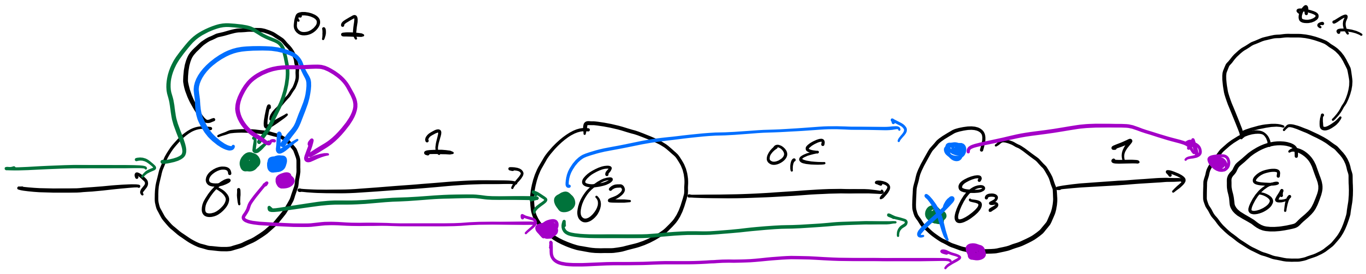
Example 2. On $\Sigma = \{0, 1\}$.

Goal: $\{ w \mid w \text{ has a '1' as the third-to-last symbol} \}$.



0, 1

Example 3.



$\{w \mid$ all strings with '11' or '101' as substrings.$\}$

101 ✓

Note: One way to do this is by tracking live branches with your fingers.

<u>Def.</u> (Power set). The power set of set Q, denoted $\mathcal{P}(Q)$, is the set of all subsets of Q.

$$Q = \{a, b, c\}$$

$$\mathcal{P}(Q) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\} = \text{size } 2^3 = 8.$$

$$|\mathcal{P}(Q)| = 2^{|Q|}$$

<u>Def. (NFA, formally.)</u> A Nondeterministic Finite Automaton (NFA) is a 5-tuple: $(Q, \Sigma, \delta, q_0, F)$, where:

Q is a finite set of states,

Σ is a finite alphabet,

q_0 is the start state,

$F \subseteq Q$ is the set of accept states,

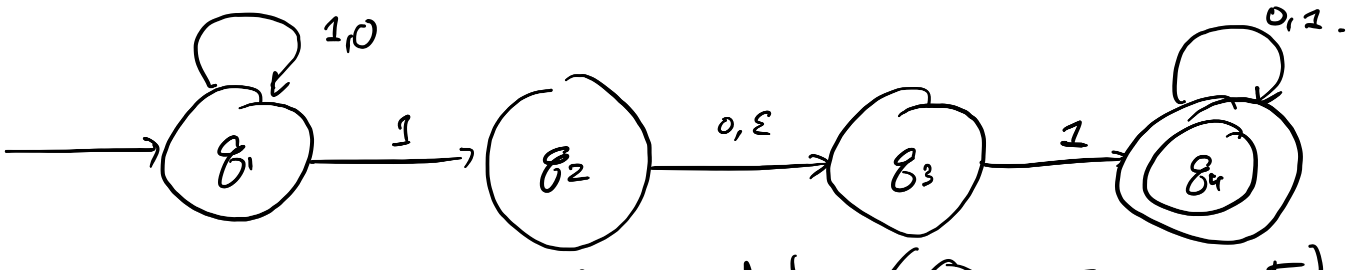
$\delta : Q \times \Sigma_\varepsilon \to \mathcal{P}(Q)$ is our transition function.

$\Sigma \cup \{\varepsilon\}$

An NFA __accepts__ a string $w = w_1 w_2 w_3 \ldots w_m$, $w_i \in \Sigma_\varepsilon$ if there exists a sequence of states $r_0, r_1, r_2, \ldots, r_m \in Q$ such that

$$r_0 = q_0$$
$$r_{i+1} \in \delta(r_i, w_{i+1}), \text{ for } i = 0, 1, \ldots, m-1,$$
$$r_m \in F.$$

Example. writing the formal defn of an NFA state diagram.



This corresponds to an NFA $N = (Q, \Sigma, \delta, q_0, F)$,

with $Q = \{q_1, q_2, q_3, q_4\}$

$\Sigma = \{0, 1\}$

$q_0 = q_1$

$F = \{q_4\}$

δ:

	0	1	ε
q_1	$\{q_1\}$	$\{q_1, q_2\}$	$\emptyset$
q_2	$\{q_3\}$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\{q_4\}$	$\emptyset$
q_4	$\{q_4\}$	$\{q_4\}$	$\emptyset$

__Next part:__ How to convert any NFA to a DFA.