COMS W3261 — Lecture 2, Part 3:

Reducing NFAs to DFAs.

We'll prove a theorem that implies:

Fact: A language is regular if and only if it is recognized by some NFA.

(reg $\longrightarrow$ DFA $\longrightarrow$ NFA)
(NFA $\longrightarrow$ DFA $\longrightarrow$ reg)

Theorem. Every NFA has an equivalent DFA.

Strategy. Our DFA will use every possible set of states in the NFA as a single state in the DFA. Our transition function will then simulate the actions of the NFA.

Proof: Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA that recognizes the language A. We'll build a DFA $M = (Q', \Sigma, \delta', q_0', F')$ that also recognizes A.

$$Q' = \mathcal{P}(Q)$$

Σ same

$F' = \{ R \in Q' \mid R \text{ contains an accept state of } N \}$.

Finally, define δ' to act like our NFA does. At each step of our NFA:

1. We start in a set of states R.
2. We read in an input symbol s, and go to all states reachable from R by s-edges
3. We go to all states reachable by ε-arrows.

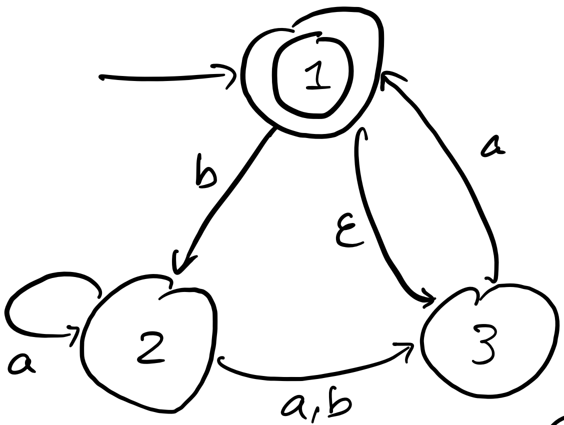
For $R \subseteq Q$, let $E(R)$ denote all states reachable from R by

ε-arrows. Define

$$\delta'(R, s) = \{q \in Q \mid q \in E(\delta(r, s)) \text{ for some } r \in R\}.$$

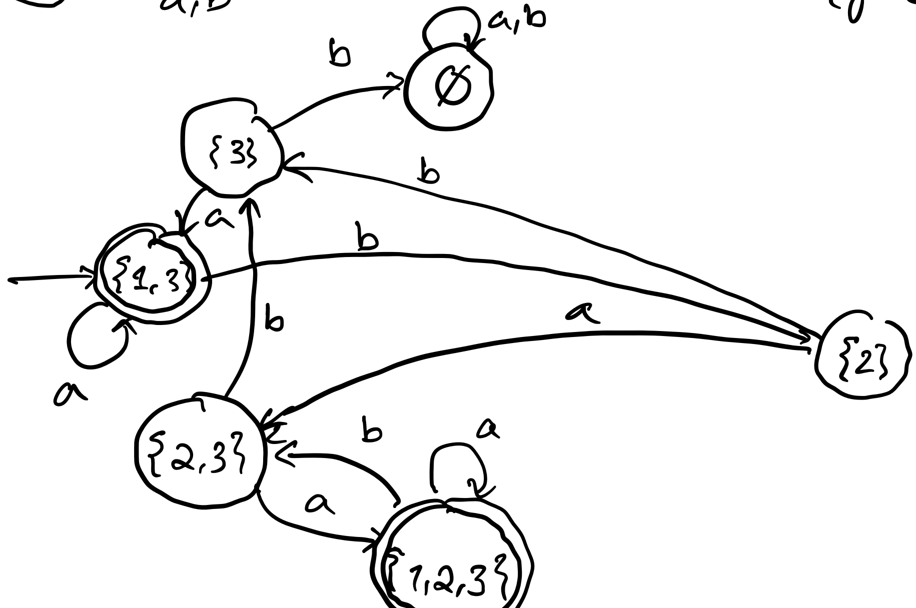
$$q_0' = E(\{q_0\})$$

Example. Converting an NFA to a DFA. $\Sigma = \{a, b\}$



$M = (Q', \Sigma, \delta', q_0', F')$.

1. $Q' = \mathcal{P}(Q)$
2. $q_0' = E(\{q_0\})$
3. $F' = \{R \in Q' \mid R$ contains an accept state of $N\}$.
4. $\delta'(R, a) = \{q \in Q \mid q \in E(\delta(r, a)), r \in R\}$



Next time:

See how to use NFAs to prove regular languages are closed under regular operations.

Reading: Sipser, end 1.1, sec. 1.2

HW #1 due Tuesday, 7/6/21 @ 11:59PM.