CONS W3261 — Lecture 3.3

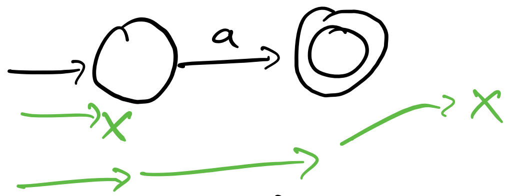
"Regular expressions are no more powerful than NFA's"

Lemma (Reg. Ex. $\longrightarrow$ NFA.) If a language is described by a regular expression, then it is regular.

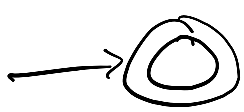
Idea: Take any generic regular expression R. We've defined R inductively, so we can build up our NFA inductively.

Proof. Let R be a regular expression. By our definition, R takes one of six forms. We show how to build an NFA equivalent to each.

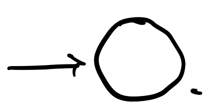
1. $R = a$, for some $a \in \Sigma$. Then $L(R) = \{a\}$, and the following NFA is equivalent.



2. $R = \varepsilon$. $L(R) = \{\varepsilon\}$.



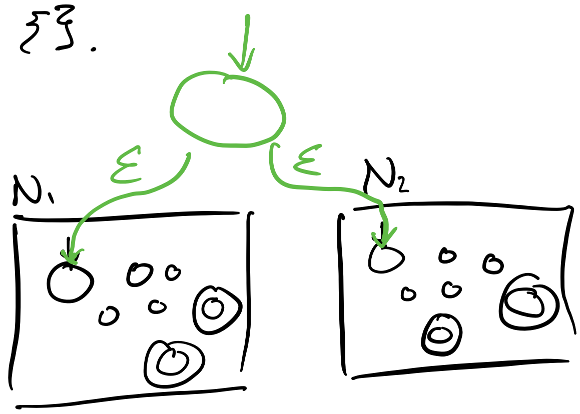
3. $R = \emptyset$. $L(R) = \emptyset, \{\}$.



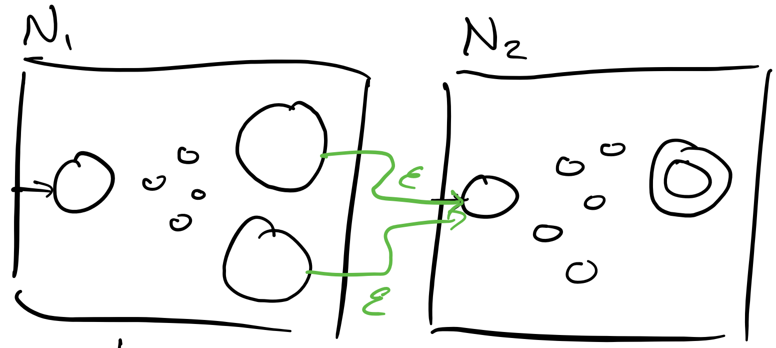
4. $R = R_1 \cup R_2$.

Let N_1 recognize R_1

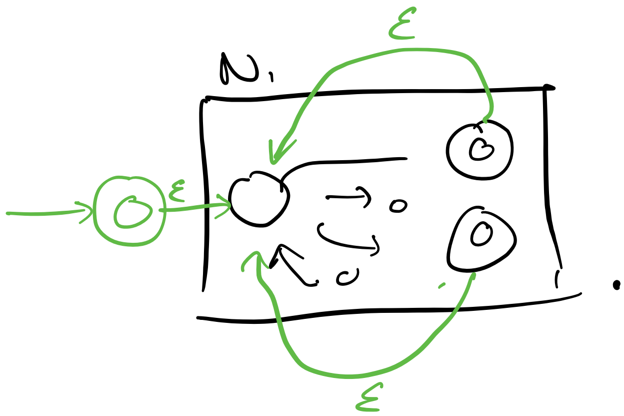
Let machine N_2 recognize R_2



5. $R = R_1 \circ R_2$. Let N_1, N_2 be NFAs recognizing R_1, R_2:

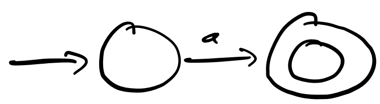


6. $R = R_1^*$. Let N_1 be an NFA recognizing R_1.

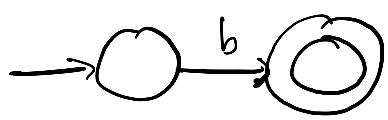


__Example.__ Convert $(a \cup b)^* aba$ to an NFA.

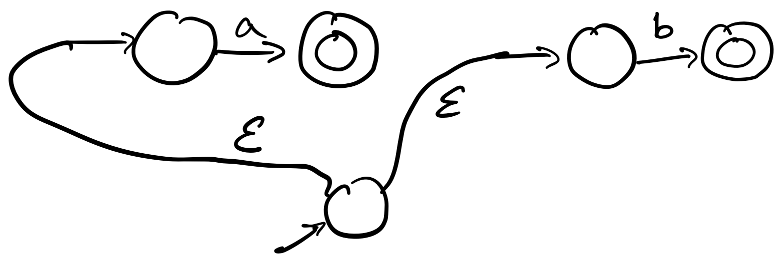
a:



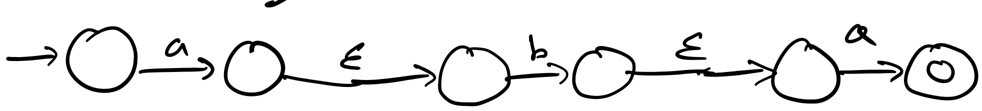
b:

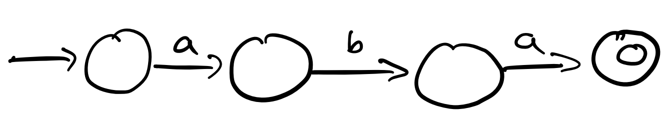


$a \cup b$:



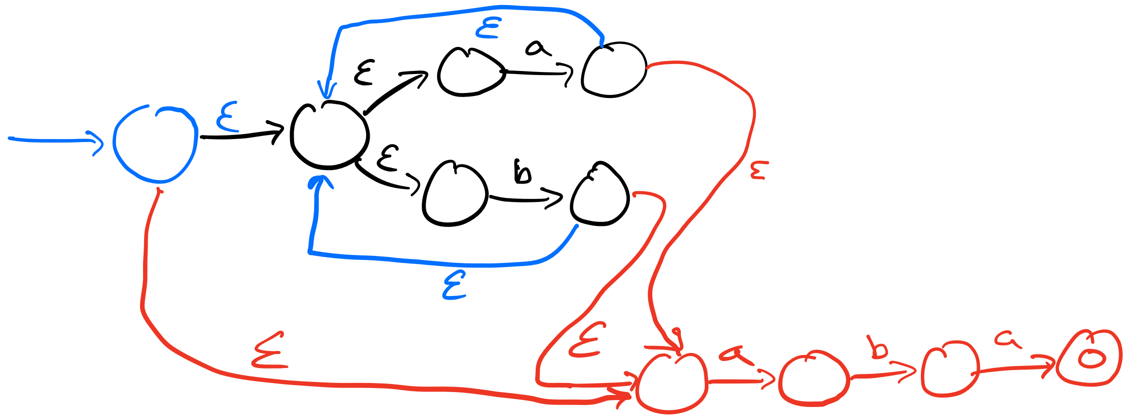
aba:





(equivalent.)

$(a \cup b)^* aba$:



HW 2: Due Monday, 7/12/21 @ 11:59PM EST

Readings: Sipser end of 1.2, 1.3.