Sipser 63-65

(Inductive) Def: A regular expression is:

base cases:
- $a \in \Sigma$ (short for $\{a\}$)
- ε (short for $\{\varepsilon\}$)
- $\emptyset$ (short for $\{\}$)

inductive steps:
- $R_1 \cup R_2$
- $R_1 R_2$ (short for $R_1 \circ R_2$)
- R_1^*

Shorthand: $R^+ := RR^*$

If $S = \{a, b, c\}$, S (as a reg exp) $:= (a \cup b \cup c)$

$R^k = \underbrace{RRR \cdots R}_{k \text{ times}}$

if $\Sigma = \{0, 1\}$, $\Sigma^2\Sigma^+$

$= \Sigma\Sigma\Sigma\Sigma^*$

$= (0 \cup 1)(0 \cup 1)(0 \cup 1)(0 \cup 1)^*$

0^*10^*
- 0^*: zero or more zeroes
- 1: concat w/ 1
- 0^*: concat w/ zero or more zeroes

$(0 \cup \varepsilon)(1 \cup \varepsilon)$

$= \{01, 0\varepsilon, \varepsilon 1, \varepsilon\varepsilon\}$

$= \{01, 0, 1, \varepsilon\}$

$(0 \cup 1)\emptyset = \emptyset$
- $(0 \cup 1)$: pick something in $\{0, 1\}$
- $\emptyset$: concat w/ something in the set $\{\}$

$\emptyset^* = \{\varepsilon\}$.

concatenate zero or more things from $\{\}$.